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Project scheduling with cost minimization and time constraint formulated as a linear programming problem

Lập tiến độ dự án với việc tối ưu hóa chi phí và ràng buộc về thời gian được thiết lập thành bài toán quy hoạch tuyến tính

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Abstract

In construction management, the task of optimizing a project schedule to achieve a minimal cost and meet a project deadline is very crucial. This study formulates this task as a linear programming problem and investigates the application of the open-source Google OR-Tools in solving such problem. A computer program has been constructed based on the formulated linear programming problem and developed with Microsoft Visual Studio and Google OR-Tools. This program has tested with a simplified project scheduling optimization task.

Keywords: Construction management; project schedule optimization; linear programming; time-cost analysis.

Tóm tắt

Trong quản lý xây dựng, nhiệm vụ tối ưu hóa tiến độ dự án để đạt được chi phí tối thiểu và thỏa mãn thời gian hoàn thành của dự án là rất quan trọng. Nghiên cứu của chúng tôi mô hình hóa vấn đề này như một bài toán quy hoạch tuyến tính và nghiên cứu ứng dụng của công cụ mã nguồn mở Google OR-Tools trong việc giải quyết vấn đề này. Một chương trình tính toán đã được xây dựng dựa trên bài toán lập trình tuyến tính và phát triển bằng Microsoft Visual Studio kết hợp với Google OR-Tools. Chương trình này đã thử nghiệm với một bài toán tối ưu hóa tiến độ dự án đơn giản.

Từ khóa: Quản lý xây dựng; tối ưu hóa tiến độ dự án; quy hoạch tuyến tính; phân tích thời gian-chi phí.

1. Introduction

A project includes a set of activities with their costs and durations. The construction industry, which is featured by constant changes in the operational environment, pressures to maintain schedules/costs with increasingly complex techniques, makes project management a highly challenging task [1-7]. Both owners and contractors have a great desire of minimizing the project cost and satisfy the project schedule. The reason contractors have a

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great risk of severe financial penalty for not completing a project on time. Addition, project owners always wish to finish the project as early as possible to put their facilities into operation.

In practice, a construction activity can be executed in several modes with different requirements of cost and time. A contractor can inspect the relationship between time and cost of each activity through cost/time estimation and historical records of similar activities performed in the past. A simple yet effective approach of time-cost modeling is to express the relationship between them as a linear function. By doing so, the task of optimizing the project schedule with cost minimization and time constraint can be formulated as a linear programming problem. Linear programming refers to a mathematical modeling approach within which a linear objective function is either minimized or maximized when subjected to a set of linear constraints [8].

This problem structure is transparent and easy to understand and this linear optimization problem is also easy to solve by existing open-source packages such as the Google OR-Tools. More importantly, the Google OR-Tools can be used within the Microsoft Visual Studio with C# programming language [9]. This fact means that sophisticated schedule optimization software program based on the Google OR-Tools and Microsoft Visual Studio can be quickly developed and used by project managers. This study exploits the aforementioned advantage of linear programming modeling, the Google OR-Tools, and the Microsoft Visual Studio with C# programming language to develop a computer program and tested it with a simplified project scheduling optimization task.

2. Problem formulation

The project schedule optimization can be formulated as the following optimization problem [10-12]:

$$\text{Minimize } \sum_{\forall i} C_i + \sum_{\forall i} S_i \qquad (1)$$

where $\sum_{\forall i} C_i$ is the activity direct cost and $\sum_{\forall i} S_i$ is the sum of the activity starting time.

Subject to

$$S_j - S_i - D_i \geq 0 \; \forall i, j \in FS_{Set} \qquad (2)$$

$$C_i = f(D_i), \; \forall i \qquad (3)$$

$$S_N + D_N \leq T_{\text{Limit}} \qquad (4)$$

Herein, the objective function is a sum of the direct activity costs and activity start times. FS_{Set} is the set of activities with finish (i) – start (j) (FS) relationship. Eq. (3) describes the linear relationship between activity time and cost. The decision variables of the problems are activity start times (S_i) and durations (D_i). Eq. (4) means that the finishing time of the last activity (activity N) must be less than a certain number (T_{Limit}).

3. Model application

In this section of the article, a simple project consisting of six activities with their duration ranges and linear cost-time functions is used to demonstrate the linear programming formulation and solving with the employment of the Google OR-Tools. This tool is implemented within the Microsoft Visual Studio and C# programming. Visual C# . NET is a general purpose and multi-paradigm programming language that facilitates fast developments of computer software used for solving problems in civil engineering [13-20]. The project information with their duration ranges and linear cost-time functions is shown in **Table 1**. The time relationships of activities are depicted in **Fig. 1**. Herein, S_2, S_3, S_4, S_5, S_6,

D_1, D_2, D_3, D_4, D_5, and T_6 are the decision variables of the optimization problem. It is noted that the activity 1's start time $S_2 = 0$.

The objective function of the problem is expressed as:

$$\text{Min. } f = S_2 + S_3 + S_4 + S_5 + S_6 + C_1 + C_2 + C_3 + C_4 + C_5 + C_6 \quad (5)$$

s.t.

$S_2 - S_1 - D_1 \geq 0$; $S_3 - S_1 - D_1 \geq 0$; $S_4 - S_1 - D_1 \geq 0$; $S_5 - S_2 - D_2 \geq 0$;

$S_5 - S_3 - D_3 \geq 0$; $S_5 - S_4 - D_4 \geq 0$; $S_6 - S_5 - D_5 \geq 0$;

$C_1 = -30D_1 + 500$; $C_2 = -25D_2 + 400$; $C_3 = -10D_3 + 600$

$C_4 = -100D_4 + 2000$; $C_5 = -50D_5 + 1500$; $C_6 = -10D_6 + 300$

$S_6 + D_6 \leq 18$

Table 1. Project information

Activity	Predecessor	Duration range	Cost-time function
1		[1, 3]	C = -30D + 500
2	1	[2, 4]	C = -25D + 400
3	1	[3, 6]	C = -10D + 600
4	1	[2, 5]	C = -100D + 2000
5	2, 3, 4	[3, 5]	C = -50D + 1500
6	5	[4, 7]	C = -10D + 300

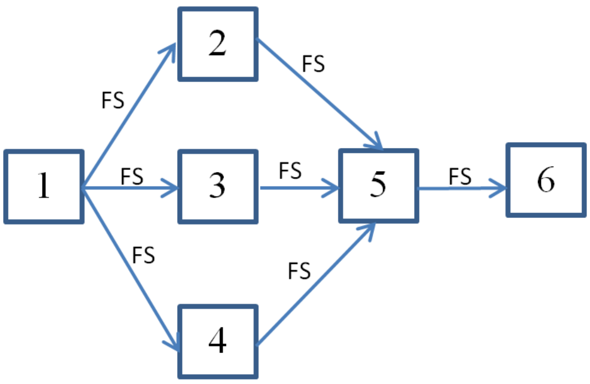

Fig. 1 Network diagram

```
Solver solver = Solver.CreateSolver("SCIP"); // For solving LP
// Create the variables which are start times of activities
int S1 = 0;
Variable S2 = solver.MakeIntVar(0, 100, "S2");
Variable S3 = solver.MakeIntVar(0, 100, "S3");
Variable S4 = solver.MakeIntVar(0, 100, "S4");
Variable S5 = solver.MakeIntVar(0, 100, "S5");
Variable S6 = solver.MakeIntVar(0, 100, "S6");
// Create the variables which are durations of activities
Variable D1 = solver.MakeIntVar(1, 3, "D1");
Variable D2 = solver.MakeIntVar(2, 4, "D2");
Variable D3 = solver.MakeIntVar(3, 6, "D3");
Variable D4 = solver.MakeIntVar(2, 5, "D4");
Variable D5 = solver.MakeIntVar(3, 5, "D5");
Variable D6 = solver.MakeIntVar(4, 7, "D6");
```

Fig. 2 Setting decision variables

```
// Set the constraints
int ProjectTimeLimit = 18; // day
solver.Add(S2 - S1 - D1 >= 0);
solver.Add(S3 - S1 - D1 >= 0);
solver.Add(S4 - S1 - D1 >= 0);
solver.Add(S5 - S2 - D2 >= 0);
solver.Add(S5 - S3 - D3 >= 0);
solver.Add(S5 - S4 - D4 >= 0);
solver.Add(S6 - S5 - D5 >= 0);
solver.Add(S6 + D6 <= ProjectTimeLimit);
// Set objective function
solver.Minimize(S2 + S3 + S4 + S5 + S6
                + (-30 * D1 + 500) + (-25 * D2 + 400)
                + (-10 * D3 + 600) + (-100 * D4 + 2000)
                + (-50 * D5 + 1500) + (-10 * D6 + 300));
solver.Solve();
```

Fig. 3 Setting problem constraints and objective function

```
// Start time of each activity
double S1_sol = S1;
double S2_sol = S2.SolutionValue();
double S3_sol = S3.SolutionValue();
double S4_sol = S4.SolutionValue();
double S5_sol = S5.SolutionValue();
double S6_sol = S6.SolutionValue();

double D1_sol = D1.SolutionValue();
double D2_sol = D2.SolutionValue();
double D3_sol = D3.SolutionValue();
double D4_sol = D4.SolutionValue();
double D5_sol = D5.SolutionValue();
double D6_sol = D6.SolutionValue();

// Finish time of each activitity = start time + duration
double F1 = S1_sol + D1_sol;
double F2 = S2_sol + D2_sol;
double F3 = S3_sol + D3_sol;
double F4 = S4_sol + D4_sol;
double F5 = S5_sol + D5_sol;
double F6 = S6_sol + D6_sol;
 Console.WriteLine("Solution:");
 Console.WriteLine("Objective value f(x,y) = " + solver.Objective().Value());
 Console.WriteLine("S1 = {0}. D1 = {1}. F1 = {2}.", S1_sol, D1_sol, F1);
 Console.WriteLine("S2 = {0}. D2 = {1}. F2 = {2}.", S2_sol, D2_sol, F2);
 Console.WriteLine("S3 = {0}. D3 = {1}. F3 = {2}.", S3_sol, D3_sol, F3);
 Console.WriteLine("S4 = {0}. D4 = {1}. F4 = {2}.", S4_sol, D4_sol, F4);
 Console.WriteLine("S5 = {0}. D5 = {1}. F5 = {2}.", S5_sol, D5_sol, F5);
 Console.WriteLine("S6 = {0}. D6 = {1}. F6 = {2}.", S6_sol, D6_sol, F6);

 Console.WriteLine("Project duration is {0} (day).", F6);

 double ProjectCost = (-30 * D1_sol + 500) + (-25 * D2_sol + 400)
                 + (-10 * D3_sol + 600) + (-100 * D4_sol + 2000)
                 + (-50 * D5_sol + 1500) + (-10 * D6_sol + 300);
 Console.WriteLine("Project cost is {0} ($).", ProjectCost);
```

Fig. 4 Display problem results

The code used for solving the aforementioned optimization process is demonstrated in **Fig. 2** (Setting decision variables) and **Fig. 3** (Setting problem constraints and objective function). The code used for displaying the optimization results are shown in **Fig. 4**. The detailed optimized schedule is reported in **Table 2**. The solution to the problem of interest is as follows:

(i) The objective function value is 4290.

(ii) The start time, duration, and finish time of each activity is as follows:

$S_1 = 0$. $D_1 = 3$. $F_1 = 3$. $S_2 = 3$. $D_2 = 4$. $F_2 = 7$.

$S_3 = 3$. $D_3 = 5$. $F_3 = 8$. $S_4 = 3$. $D_4 = 5$. $F_4 = 8$.

$S_5 = 8$. $D_5 = 5$. $F_5 = 13$. $S_6 = 13$. $D_6 = 5$. $F_6 = 18$.

(iii) The project duration is 18 (day).

(iv) The project cost is $4260.

Table 2. Project scheduling results

Activity	Start time	Optimal duration (day)	Finish time
1	0	3	3
2	3	4	7
3	3	5	8
4	3	5	8
5	8	5	13
6	13	5	18

4. Concluding remarks

This study develops a software program based on linear programming, Google OR-Tools, and Microsoft Visual Studio with C# to perform construction project schedule optimization. The program is able to compute the project schedule automatically with a minimal activity cost and a total project duration that satisfies a pre-specified time constraint. Future extensions of the current work may include the consideration of project indirect cost and the integration the current linear programming formulation into other sophisticated scheduling optimization models such as resource leveling [1, 21, 22], time-cost trade-off [23-27], labor utilization [2, 28], etc.

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